

AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

1 1. (Currently amended) A method for using a computer system to solve an
2 unconstrained interval global optimization problem specified by a function f ,
3 wherein f is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots x_n)$, the method
4 comprising:

5 receiving a representation of the function f at the computer system;
6 storing the representation in a memory within the computer system; and
7 performing an interval global optimization process using interval
8 operations to compute guaranteed bounds on a globally minimum value of the
9 function $f(\mathbf{x})$;

10 wherein performing the interval global optimization process involves,
11 applying term consistency over a subbox \mathbf{X} , and
12 excluding any portion of the subbox \mathbf{X} that violates term
13 consistency.

1 2. (Original) The method of claim 1, wherein applying term consistency
2 involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a first term, $g(\mathbf{x}')$, thereby producing a modified equation $g(\mathbf{x}') = h(\mathbf{x})$,
5 wherein the first term $g(\mathbf{x}')$ can be analytically inverted to produce an inverse
6 function $g^{-1}(\mathbf{y})$;

7 substituting the subbox \mathbf{X} into the modified equation to produce the
8 equation $g(\mathbf{X}') = h(\mathbf{X})$;
9 solving for $\mathbf{X}' = g^{-1}(h(\mathbf{X}))$; and
10 intersecting \mathbf{X}' with the subbox \mathbf{X} to produce a new subbox \mathbf{X}^+ ;
11 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
12 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
13 the size of the subbox \mathbf{X} .

1 3. (Original) The method of claim 1, wherein performing the interval
2 global optimization process involves:
3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$;
4 removing from consideration any subbox for which $f(\mathbf{x}) > f_bar$;
5 applying term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} ;
6 and
7 excluding any portion of the subbox \mathbf{X} that violates the inequality.

1 4. (Original) The method of claim 1, wherein performing the interval
2 global optimization process involves:
3 determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
4 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
5 removing from consideration any subbox for which $\mathbf{g}(\mathbf{x})$ is bounded away
6 from zero, thereby indicating that the subbox does not include a global minimum
7 of $f(\mathbf{x})$; and
8 applying term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=0$
9 over the subbox \mathbf{X} ; and
10 excluding any portion of the subbox \mathbf{X} that violates a component.

1 5. (Currently amended) The method of claim 1, wherein performing the
2 interval global optimization process involves:
3 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
4 function $f(\mathbf{x})$;
5 removing from consideration any subbox for which a diagonal element of
6 the Hessian is always negative, which indicates that the f is not convex and
7 consequently does not contain a global minimum within the subbox;
8 applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the
9 subbox \mathbf{X} ; and
10 ~~excluding any portion of the subbox \mathbf{X} that violates an inequality~~
11 these inequalities.

1 6. (Currently amended) The method of claim 1, wherein performing the
2 interval global optimization process involves performing the Newton method,
3 wherein performing the Newton method involves:
4 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the function f evaluated as a function of
5 a point \mathbf{x} over the subbox \mathbf{X} ;
6 computing an approximate inverse \mathbf{B} of the center of $\mathbf{J}(\mathbf{x}, \mathbf{X})$; and
7 using the approximate inverse \mathbf{B} to analytically determine the system
8 $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes
9 components $g_i(\mathbf{x})$ ($i=1, \dots, n$); ~~and~~
10 ~~applying term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for~~
11 ~~each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} ; and~~
12 ~~excluding any portion of the subbox \mathbf{X} that violates a component.~~

1 7. (Original) The method of claim 1, further comprising terminating
2 attempts to further reduce the subbox \mathbf{X} when:
3 the width of \mathbf{X} is less than a first threshold value; and

4 the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

1 8. (Currently amended) A computer-readable storage medium storing
2 instructions that when executed by a computer cause the computer to perform a
3 method for using a computer system to solve an unconstrained interval global
4 optimization problem specified by a function f , wherein f is a scalar function of a
5 vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the method comprising:
6 receiving a representation of the function f at the computer system;
7 storing the representation in a memory within the computer system; and
8 performing an interval global optimization process using interval
9 operations to compute guaranteed bounds on a globally minimum value of the
10 function $f(\mathbf{x})$;
11 wherein performing the interval global optimization process involves,
12 applying term consistency over a subbox \mathbf{X} , and
13 excluding any portion of the subbox \mathbf{X} that violates term
14 consistency.

1 9. (Original) The computer-readable storage medium of claim 8, wherein
2 applying term consistency involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a first term, $g(\mathbf{x}')$, thereby producing a modified equation $g(\mathbf{x}') = h(\mathbf{x})$,
5 wherein the first term $g(\mathbf{x}')$ can be analytically inverted to produce an inverse
6 function $g^{-1}(\mathbf{y})$;
7 substituting the subbox \mathbf{X} into the modified equation to produce the
8 equation $g(\mathbf{X}') = h(\mathbf{X})$;
9 solving for $\mathbf{X}' = g^{-1}(h(\mathbf{X}))$; and
10 intersecting \mathbf{X}' with the subbox \mathbf{X} to produce a new subbox \mathbf{X}^+ ;

11 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
12 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
13 the size of the subbox \mathbf{X} .

1 10. (Original) The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves:
3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$;
4 removing from consideration any subbox for which $f(\mathbf{x}) > f_bar$;
5 applying term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} ;
6 and
7 excluding any portion of the subbox \mathbf{X} that violates the inequality.

1 11. (Original) The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves:
3 determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
4 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
5 removing from consideration any subbox for which $\mathbf{g}(\mathbf{x})$ is bounded away
6 from zero, thereby indicating that the subbox does not include a global minimum
7 of $f(\mathbf{x})$; and
8 applying term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=0$
9 over the subbox \mathbf{X} ; and
10 excluding any portion of the subbox \mathbf{X} that violates a component.

1 12. (Currently amended) The computer-readable storage medium of claim
2 8, wherein performing the interval global optimization process involves:
3 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
4 function $f(\mathbf{x})$;

5 removing from consideration any subbox for which a diagonal element of
6 the Hessian is always negative, which indicates that the f is not convex and
7 consequently does not contain a global minimum within the subbox;
8 applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the
9 subbox \mathbf{X} ; and
10 excluding any portion of the subbox \mathbf{X} that ~~violates an inequality~~ violate
11 these inequalities.

1 13. (Currently amended) The computer-readable storage medium of claim
2 8, wherein performing the interval global optimization process involves
3 performing the Newton method, wherein performing the Newton method
4 involves:
5 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the function f evaluated as a function of
6 a point \mathbf{x} over the subbox \mathbf{X} ;
7 computing an approximate inverse \mathbf{B} of the center of $\mathbf{J}(\mathbf{x}, \mathbf{X})$; and
8 using the approximate inverse \mathbf{B} to analytically determine the system
9 $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes
10 components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
11 ~~applying term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for~~
12 ~~each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} ; and~~
13 ~~excluding any portion of the subbox \mathbf{X} that violates a component.~~

1 14. (Original) The computer-readable storage medium of claim 8, wherein
2 the method further comprises terminating attempts to further reduce the subbox \mathbf{X}
3 when:
4 the width of \mathbf{X} is less than a first threshold value; and
5 the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

1 15. (Currently amended) An apparatus that solves an unconstrained
2 interval global optimization problem specified by a function f , wherein f is a scalar
3 function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the apparatus comprising:
4 a receiving mechanism that is configured to receive a representation of the
5 function f ;
6 a memory for storing the representation; and
7 an interval global optimization mechanism that is configured to perform
8 an interval global optimization process using interval operations to compute
9 guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$;
10 a term consistency mechanism within the interval global optimization
11 mechanism that is configured to,
12 apply term consistency over a subbox \mathbf{X} , and to
13 exclude any portion of the subbox \mathbf{X} that violates term
14 consistency.

1 16. (Original) The apparatus of claim 15, wherein the term consistency
2 mechanism includes:
3 a symbolic manipulation mechanism that is configured to symbolically
4 manipulate an equation within the computer system to solve for a first term, $g(\mathbf{x}')$,
5 thereby producing a modified equation $g(\mathbf{x}') = h(\mathbf{x})$, wherein the first term $g(\mathbf{x}')$
6 can be analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;
7 a solving mechanism that is configured to,
8 substitute the subbox \mathbf{X} into the modified equation to
9 produce the equation $g(\mathbf{X}') = h(\mathbf{X})$, and to
10 solve for $\mathbf{X}' = g^{-1}(h(\mathbf{X}))$; and
11 an intersecting mechanism that is configured to intersect \mathbf{X}' with the
12 subbox \mathbf{X} to produce a new subbox \mathbf{X}^+ ;

13 wherein the new subbox X^+ contains all solutions of the equation within
14 the subbox X , and wherein the size of the new subbox X^+ is less than or equal to
15 the size of the subbox X .

1 17. (Original) The apparatus of claim 15,
2 wherein the interval global optimization mechanism is configured to,
3 keep track of a least upper bound f_bar of the function $f(x)$,
4 and to
5 remove from consideration any subbox for which
6 $f(x) > f_bar$;
7 wherein the term consistency mechanism is configured to,
8 apply term consistency to the inequality $f(x) \leq f_bar$ over
9 the subbox X , and to
10 exclude any portion of the subbox X that violates the
11 inequality.

1 18. (Original) The apparatus of claim 15,
2 wherein the interval global optimization mechanism is configured to,
3 determine a gradient $g(x)$ of the function $f(x)$, wherein $g(x)$
4 includes components $g_i(x)$ ($i=1, \dots, n$), and to
5 remove from consideration any subbox for which $g(x)$ is
6 bounded away from zero, thereby indicating that the subbox does
7 not include a global minimum of $f(x)$; and
8 wherein the term consistency mechanism is configured to,
9 apply term consistency to each component $g_i(x)=0$
10 ($i=1, \dots, n$) of $g(x)=0$ over the subbox X , and to
11 exclude any portion of the subbox X that violates a
12 component.

1 19. (Currently amended) The apparatus of claim 15,
2 wherein the interval global optimization mechanism is configured to,
3 determine diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the
4 Hessian of the function $f(\mathbf{x})$, and to
5 remove from consideration any subbox for which a
6 diagonal element of the Hessian is always negative, which
7 indicates that the f is not convex and consequently does not contain
8 a global minimum within the subbox; and
9 wherein the term consistency mechanism is configured to,
10 apply term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$
11 ($i=1, \dots, n$) over the subbox \mathbf{X} , and to
12 exclude any portion of the subbox \mathbf{X} that violates an
13 ~~inequality~~ inequality ~~violate these inequalities.~~

1 20. (Currently amended) The apparatus of claim 15, further comprising a
2 Newton mechanism within the interval global optimization mechanism that is
3 configured to:
4 compute the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the function f evaluated as a function of a
5 point \mathbf{x} over the subbox \mathbf{X} ;
6 compute an approximate inverse \mathbf{B} of the center of $\mathbf{J}(\mathbf{x}, \mathbf{X})$; and to
7 using the approximate inverse \mathbf{B} to analytically determine the system
8 $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes
9 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
10 ~~wherein the term consistency mechanism is configured to,~~
11 ~~apply term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$~~
12 ~~$(i=1, \dots, n)$ for each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} , and to~~
13 ~~exclude any portion of the subbox \mathbf{X} that violates term~~
14 ~~consistency.~~

1 21. (Original) The apparatus of claim 15, further comprising a termination
2 mechanism that is configured to terminate attempts to further reduce the subbox **X**
3 when:
4 the width of **X** is less than a first threshold value; and
5 the magnitude of $f(\mathbf{X})$ is less than a second threshold value.